

# Quark Confinement: The Hard Problem of Hadron Physics

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## Abstract.

We give a brief overview of the problem of quark confinement in hadronic physics, and outline a few of the suggested explanations of the confining force.

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## 1. Introduction

The hadron spectrum found in nature consists of color singlet combinations of color non-singlet objects: the quarks and gluons. Unlike atomic physics, where electrons can readily be separated from atoms, there is no color-charge version of ionization in hadronic physics. Every attempt to kick a quark free from a hadron, via high-energy collisions, only results in the production of more color-singlet hadrons; a non-singlet particle is never produced. Particle and nuclear physicists have become accustomed to this fact, which is often referred to as “color confinement”, but after thirty-four years of intense effort this very basic feature of hadron physics still has no generally agreed upon explanation. Color confinement is therefore a hard problem. In this article we would like to discuss some aspects of this problem which we think are important, and to briefly survey a few of the main avenues of research.

To begin with, what would be the energy of an isolated quark? In gauge theories, abelian or non-abelian, a charge density,  $\rho_{quark}^a$ , is the source of a longitudinal electric field, as required by the Gauss Law

$$\vec{\nabla} \cdot \vec{E}^a = \rho_{quark}^a - g f^{abc} A_k^b E_k^c \quad (1)$$

where the term containing the structure constant of the gauge group  $f^{abc}$  and the gauge field  $\vec{A}^a$  is only present in non-abelian gauge theories and reflects the non-vanishing color electric charge of the gluons. Their charge is in the **8** representation of the SU(3) gauge group, and cannot neutralize the color charge of a quark in the **3** representation. So the color electric field of an isolated quark could only end on another isolated quark, or else extend out to infinity. The fact that isolated quarks are not seen in nature means that the energy stored in the associated color electric field must be very large. But *how* large? Suppose we try to

free a quark from a hadron by hitting it with a high energy (real or virtual) photon. As the struck quark begins to move away from the other quarks in the hadron, it brings along the color electric field necessary to satisfy the non-abelian Gauss Law. If the energy stored in the color electric field becomes large enough, then the system is unstable to light quark-antiquark pair creation. The antiquark of the pair binds to the struck quark, resulting in a color singlet, and the quark of the pair binds to the remaining quarks of the hadron, forming another color singlet. The two color singlet hadrons are generally still in highly excited states, and decay into lighter hadrons. The end result is a shower of ordinary hadrons, rather than a free quark and a color-ionized hadron.

So a hadron scattering experiment will not answer our question about the energy stored in the color electric field of a free quark, at least not directly. Our knowledge about this energy is therefore indirect, and comes from two sources: numerical simulations, and a pattern in the hadron spectrum known as Regge trajectories. Let us imagine “dialing” the bare quark mass parameters in the QCD Lagrangian so that all quarks are very heavy; so heavy, in fact, that pair creation processes do not become important until quark separations reach macroscopic (or even cosmic!) distances. Then, starting with a tightly bound color singlet object such as a meson, and measuring the energy required to slowly separate the massive constituent quark and antiquark by a distance  $R$ , we get an estimate for the static quark potential  $V(R)$ , and this is essentially a measure of the energy stored in the color electric field due to the quarks. Of course, in nature the current quark masses are whatever they are, and cannot be changed, but on a computer anything is possible: Nothing prevents us from simulating a version of QCD with very massive quarks, as we will discuss in more detail below.

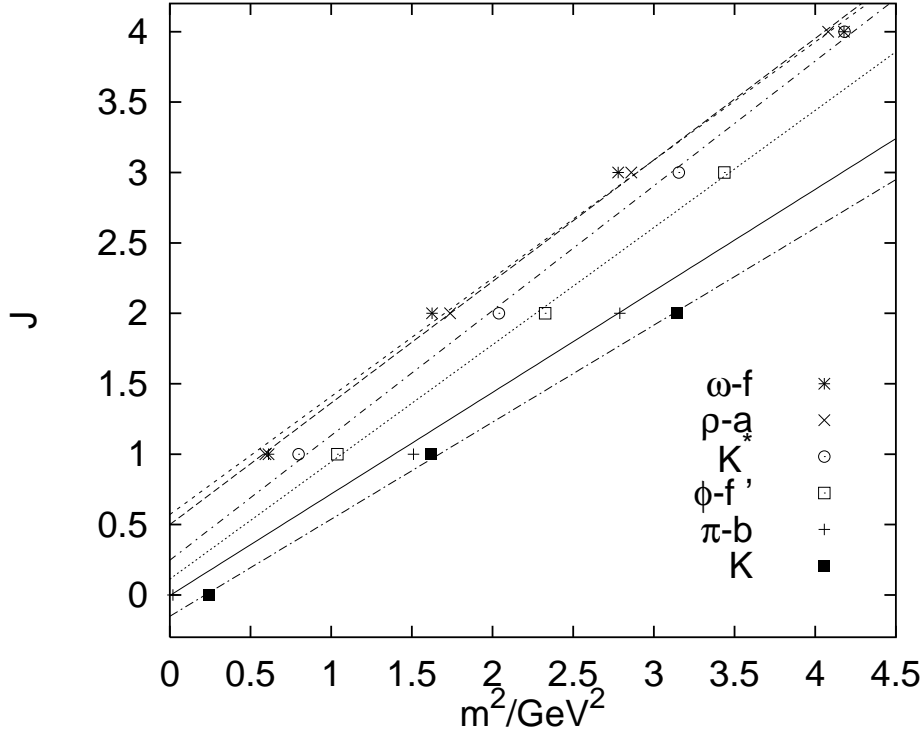
Our second source of information about the static quark potential is derived from the actual hadron spectrum. In the spectrum there exist certain metastable states which are sufficiently long-lived to show up as resonances in scattering cross-sections. The fact that it takes some time for these metastable states to decay via quark pair creation means that, for the short period prior to decay, the resonances are sensitive to interquark forces in the absence of light quark pair creation. From the masses of the resonant states, we can therefore learn a great deal about states with comparatively large quark separations, and about the energy which is stored in the associated color electric fields.

## 2. The Linear Potential

The following theorem [1] can be proven in lattice gauge theory: the force between a static quark and antiquark is always attractive but cannot increase with distance, *i.e.*

$$\frac{\partial V}{\partial R} > 0, \quad \frac{\partial^2 V}{\partial R^2} \leq 0. \quad (2)$$

The second inequality is saturated by a linear potential; the static quark potential can rise no faster than linearly with distance. The theorem does not tell us that the static quark potential actually *does* rise linearly, but hadron phenomenology suggests, and computer simulations convincingly demonstrate, that this is the true, or at least very close to the true, behavior of the potential at large quark separations.



**Figure 1.** Regge trajectories for the low-lying mesons (adapted from ref. [2]).

### 2.1. Regge Trajectories and the Spinning Stick Model

A remarkable pattern emerges in the hadronic spectrum, when the spin of mesons (and baryons) is plotted against their squared mass, as shown in Fig. 1. In such plots the mesons and baryons of given flavor quantum numbers seem to lie on nearly parallel straight lines, known as linear Regge trajectories. This is a very striking feature of the hadronic spectrum, nothing similar is found in the electroweak theory, and the question is why it occurs.

Suppose that we picture a meson as a straight line of length  $L = 2R$ , with mass per unit length  $\sigma$ . The line rotates about a perpendicular axis through its midpoint, such that the endpoints of the line are moving at the speed of light,  $v(R) = c = 1$ . Then for the energy in the rest frame, *i.e.* the mass, of the spinning stick we have

$$m = \text{Energy} = 2 \int_0^R \frac{\sigma dr}{\sqrt{1-v^2(r)}} = 2 \int_0^R \frac{\sigma dr}{\sqrt{1-r^2/R^2}} = \pi\sigma R, \quad (3)$$

and for the angular momentum

$$J = 2 \int_0^R \frac{\sigma r v(r) dr}{\sqrt{1-v^2(r)}} = \frac{2}{R} \int_0^R \frac{\sigma r^2 dr}{\sqrt{1-r^2/R^2}} = \frac{1}{2} \pi\sigma R^2. \quad (4)$$

Comparing the two expressions, we see that

$$J = \frac{1}{2\pi\sigma} m^2 = \alpha' m^2 \quad (5)$$

The constant  $\alpha'$  is known as the “Regge slope”.

From the data one estimates  $\alpha' = 1/(2\pi\sigma) = 0.9 \text{ GeV}^{-2}$ , which gives a mass/unit length of the string, or “*string tension*”, of

$$\sigma \approx 0.18 \text{ GeV}^2 \approx 0.9 \text{ GeV/fm}. \quad (6)$$

The spinning stick model is, of course, only a caricature of the real situation. In fact the various Regge trajectories do not pass through the origin, and have slightly different slopes. To make the model more realistic, one might want to relax the requirement of rigidity, and allow the “stick” to fluctuate in transverse directions. This line of thought leads to string theory. However, since QCD is the theory of quarks and gluons, the question to be answered is how a stick-like or string-like object actually emerges from that theory.

One possible answer is via the formation of a color electric flux tube. We imagine that the color electric field running between a static quark and antiquark is, for some reason, squeezed into a cylindrical region, whose cross-sectional area is nearly constant as quark-antiquark separation  $L$  increases. In that case, the energy stored in the color electric field will grow linearly with quark separation, *i.e.*

$$\text{Energy} = \sigma L \quad \text{with} \quad \sigma = \int d^2x_{\perp} \frac{1}{2} \vec{E}^a \cdot \vec{E}^a \quad (7)$$

where the integration is over a cross-section of the flux tube. This means that there will be a linearly rising potential energy associated with static sources (the “static quark potential”), and an infinite energy is required to separate these charges an infinite distance.

In this way the pattern of metastable states in the hadron spectrum suggests a picture of how the color electric field energy, in the absence of light quark pair creation, would grow with quark separation.

## 2.2. Wilson Loops and Lattice Simulations

The most reliable evidence we have about the static quark potential is obtained from computer simulations of quantum chromodynamics. For this purpose it is useful to simulate a version of QCD in which the quarks are very massive, and pair creation in the vacuum can be ignored.

Let  $Q(t)$  be the creation operator of a state at time  $t$  containing a very massive quark and a very massive antiquark, separated by a distance  $R$ . There are many operators of that sort, but, unless we fix a gauge, it is necessary for  $Q$  to be gauge-invariant. If not, then  $Q$  and correlators of  $Q$  will simply average to zero in the functional integral over gauge fields  $A$  and the quark fields  $\psi$ . Consider the unequal-times correlator

$$\langle Q^{\dagger}(T)Q(0) \rangle = \frac{1}{Z} \int DAD\psi D\bar{\psi} Q^{\dagger}(T)Q(0) e^{iS} = \langle \Psi_0 | Q^{\dagger} e^{-i(H-E_0)T} Q | \Psi_0 \rangle \quad (8)$$

where  $H$  is the Hamiltonian operator,  $E_0$  is the vacuum energy and  $\Psi_0$  is the vacuum state, in any gauge (the gauge choice does not matter if  $Q$  is gauge invariant). By transforming the theory from Minkowski space to Euclidean space by a Wick rotation of the time coordinate  $t \rightarrow it$ , and inserting a complete set of energy eigenstates  $\{\Psi_n\}$  with the quantum numbers of the heavy quark-antiquark pair, the above expression becomes

$$\begin{aligned}
\langle Q^\dagger(T)Q(0) \rangle_{\text{euclid}} &= \frac{1}{Z} \int DAD\psi D\bar{\psi} Q^\dagger(T)Q(0) e^{-S} \\
&= \langle \Psi_0 | Q^\dagger e^{-(H-\mathcal{E}_0)T} Q | \Psi_0 \rangle \\
&= \sum_n \left| \langle \Psi_0 | Q^\dagger | \Psi_n \rangle \right|^2 e^{-E_n T}
\end{aligned} \tag{9}$$

where  $E_n$  is the excitation energy (above  $\mathcal{E}_0$ ) of the energy eigenstate  $\Psi_n$ . The notation  $\langle \rangle_{\text{euclid}}$  indicates Euclidean-time expectation values, but from here on the “euclid” subscript will be dropped. We see that at large Euclidean time separations, the correlator is dominated by the minimum energy state of the Minkowski theory. So to find the *minimum* energy possible for a state containing a heavy quark antiquark pair, satisfying the Gauss Law, we have to calculate

$$E_{\min}^{q\bar{q}}(R) = - \lim_{T \rightarrow \infty} \frac{d}{dT} \log \left[ \langle Q^\dagger(T)Q(0) \rangle \right]. \tag{10}$$

At large  $T$  any choice of  $Q$  will do, providing  $Q$  is gauge-invariant, and the quarks are created at separation  $R$ . The simplest choice is

$$Q = \bar{\psi}(0) P \exp \left[ i \int_0^R dx^\mu A_\mu \right] \psi(R) \tag{11}$$

where the expression between the quark operators is a path-ordered exponential of the matrix-valued A-field, which lies in the Lie algebra of the gauge group. If the quarks are very heavy and quark loops can be ignored, then the functional integration over the quark fields can be carried out explicitly, with the result

$$\langle Q^\dagger(T)Q(0) \rangle = \kappa M^{-2T} \left\langle \text{Tr} \left[ P e^{i \oint_C dx^\mu A_\mu} \right] \right\rangle. \tag{12}$$

In this expression  $\kappa$  is a numerical prefactor, coming from a trace over Dirac matrices, and  $M$  is a constant which depends on the bare quark mass (and the UV regulator). Neither of these terms have any sensitivity to the quark separation  $R$ . The term we are interested in is the remaining expectation value, which involves only the gluon field

$$W(C) = \left\langle \text{Tr} \left[ P e^{i \oint_C dx^\mu A_\mu} \right] \right\rangle \tag{13}$$

where in this case the line integral runs around a closed rectangular contour  $C$  of length  $T$  and width  $R$ . The path-ordered exponential of such line integrals are known as *Wilson loops*. For a rectangular contour, we will denote the expectation value by  $W(R, T)$ , and the part of the potential which depends on  $R$  is extracted from

$$V(R) = - \lim_{T \rightarrow \infty} \frac{d}{dT} W(R, T). \tag{14}$$

From now on this will serve as our definition of the heavy quark potential.

Nobody knows how to calculate  $W(R, T)$  analytically in QCD, when  $R$  and  $T$  are large compared to the length scale set by the fundamental scale of QCD,  $\Lambda_{\text{QCD}}$ . However, this quantity can be calculated in regions of interest by numerical simulations, which require regulating QCD on a finite lattice. Lattice gauge theory is explained in detail in a number of texts, *e.g.* see ref. [3], but very briefly the idea is this: Continuous spacetime is replaced

by a  $D=4$  dimensional hypercubic lattice; the points on the lattice are called “sites”, the lines between neighboring sites are “links”, and the squares bounded by four neighboring links are called “plaquettes”. The Lie algebra-valued gauge field  $A_\mu(x)$  of the continuum theory is replaced by a set of group-valued link variables  $U_\mu(x)$ , associated with links of the lattice. A Wilson loop is simply the trace of the product of link variables around a closed contour on the lattice, with the understanding that when the contour runs through a link in the negative  $x$ ,  $y$ ,  $z$ , or  $t$  directions, then the hermitian conjugate  $U_\mu^\dagger(x)$  is used in place of  $U_\mu(x)$ . The lattice version of the functional integral over gauge fields,

$$Z = \int DU e^{S_W}, \quad (15)$$

is based on the “Wilson action”  $S_W$

$$S_W = \frac{\beta}{3} \sum_x \sum_{\mu=1}^3 \sum_{\nu > \mu} \text{Tr} \left[ U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x) \right] + \text{c.c.} \quad (16)$$

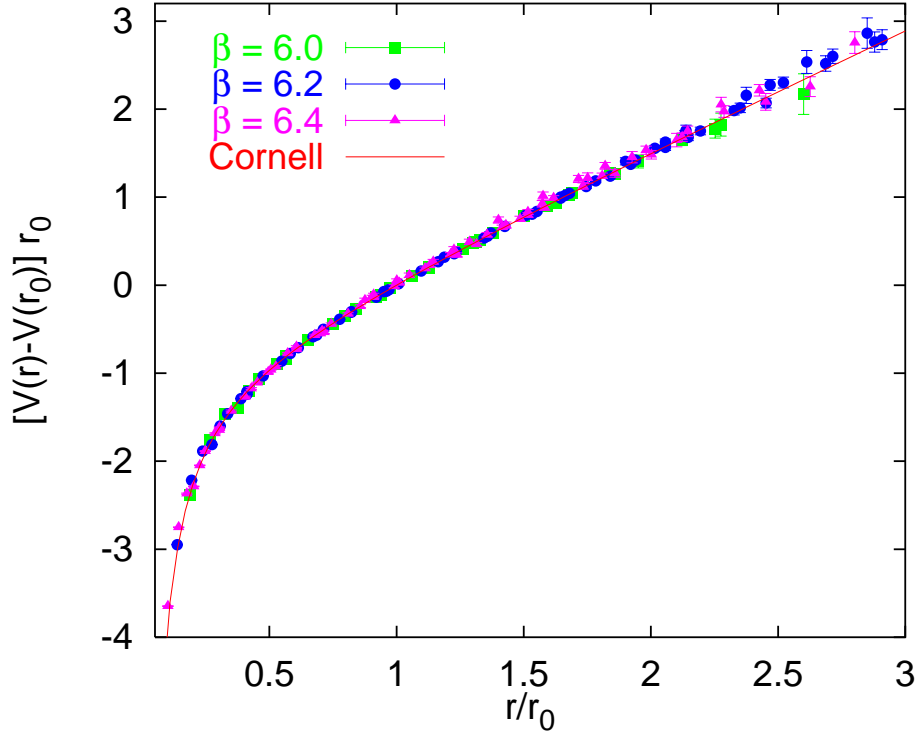
Numerical (“lattice Monte Carlo”) simulations involve stochastically generating a set of lattice gauge field configurations according to the probability distribution  $P[U] = e^{S_W} / Z$ ; an estimate of the vacuum expectation value of some operator (such as a Wilson loop) is simply the average value that the operator takes in the set. Every quantity calculated on the lattice is calculated in units in which the lattice spacing  $a = 1$ . To convert to physical units,  $a$  must be assigned a value in, say, fermis, and all lattice results are a function of the lattice spacing and the coupling  $\beta = 2N/g^2$  for the gauge group  $SU(N)$ . Of course, the masses of hadrons should not depend on the lattice spacing, but renormalization theory teaches us that at sufficiently small couplings, a change in  $a$  can be compensated for by a change in  $g$ , leaving the spectrum and other physical quantities invariant.

An example of the static quark potential obtained by lattice Monte Carlo techniques is shown in Fig. 2. Here the potential, computed at several  $\beta$  values, is plotted against quark separation in units of a certain physical scale  $r_0$ , which is about 0.5 fm. In this graph we see very convincing confirmation of the linearity of the static quark potential at large distances. If the potential rises linearly indefinitely, then the energy of an isolated quark would be infinite. It is no wonder, then, that color non-singlet particles are not produced in hadronic collisions.

The linear rise of the static quark potential at arbitrarily large quark separations is a rather astonishing and important fact, and the question — the quark confinement problem — is how to account for such behavior. Until that question is answered satisfactorily, we do not really understand hadronic physics, nor do we understand the dynamics of non-abelian gauge theories at large distance scales.

### 2.3. Further Properties of the Linear Potential

In addition to varying the quark mass in numerical simulations, one can also vary the color group representation of the “quarks” (i.e. heavy static color sources), and study the effect on the static quark potential. Numerical simulations, and some general arguments, indicate that there are two distinct sorts of representation dependence, depending on the static source separation:



**Figure 2.** The static quark potential, obtained from lattice Monte Carlo simulations, in SU(3) gauge theory. Both potential and quark separation are in units of the “Sommer scale”  $r_0 \approx 0.5 fm$  (adapted from ref. [2]).

- (i) **Casimir Scaling.** Initially the slope of the linear potential — the string tension  $\sigma$  — is proportional to the quadratic Casimir of the group representation.
- (ii) **N-ality Dependence.** Asymptotically, the force between charged fields in an SU(N) gauge theory depends only on the so-called “N-ality” of the group representation, given by the number of boxes mod  $N$  in the Young tableau of the representation.

Both of these dependencies have been observed in numerical simulations [4, 5]; there are also rather convincing arguments, based on energetics, for N-ality dependence at large distances.

We have already mentioned that the color electric field between quarks is collimated into a flux tube; this precludes long-range van der Waals forces or color dipole fields. In addition, string theory models of hadrons predict a universal, coupling-constant independent correction to the static quark potential [6]

$$V_{string}(R) = -\frac{\pi}{12R} . \quad (17)$$

There is evidence, again from numerical simulations, for the existence of this small correction to the linear potential, as well as a spectrum of excitations of the confining electric flux tube [7].

Taken together, these features of the static quark potential are quite restrictive; a completely satisfactory theory of confinement should account for all of them.

### 3. Theories of Quark Confinement

There are a number of different approaches to the quark confinement problem. Probably the most popular idea is that the QCD functional integral is dominated by some special class of field configurations which cause the expectation value of a large Wilson loop to fall off exponentially with the minimal area of the loop, i.e.  $W(C) \sim \exp[-\sigma A(C)]$ . For large rectangular loops, this behavior implies a linear static quark potential, with string tension  $\sigma$ . The leading candidates for these special configurations are magnetic monopoles and center vortices, although other objects such as merons [9], and calorons [10, 11] have also been advocated. A different approach is based on the special properties of quantization in Coulomb gauge, as we will describe briefly below. Another idea is to try to solve non-perturbatively for quark and gluon propagators and vertex functions, analytically by an infrared expansion of the complete set of Schwinger-Dyson equations, and numerically by solving a truncated set of these equations. Finally, there is a fascinating relationship between gauge theory in  $D = 4$  dimensions and string theory quantized in a special ten-dimension background geometry known as anti-DeSitter space. This is the AdS-CFT correspondence

Each of these ideas have been the subject of intense study, and the most we can do here is to give a brief indication of what they are all about. For some of these scenarios, surprising and interesting relations between them have been discovered, and some of those will be mentioned along the way. For a more detailed discussion of material in sections 3.1-3.3 below, see the review article by one of us [12].

#### 3.1. Magnetic Monopoles and Dual Superconductivity

The linear static potential would be explained if we could understand why the color electric field, between a quark and an antiquark, should be collimated into a cylindrical region – a flux tube – of constant or nearly constant cross-section. There is a very suggestive example in low temperature physics known as the Meissner effect: magnetic fields in type II superconductors are in fact collimated into magnetic flux tubes, known as Abrikosov vortices. If magnetic monopoles existed in nature, and a monopole-antimonopole pair were placed in a type II superconductor, the monopoles would be connected by a magnetic flux tube, and energy stored in the magnetic field would grow linearly with monopole separation. This example led 't Hooft [13] and Mandelstam [14] to suggest that the QCD vacuum is a “dual superconductor”, the word “dual” in this case meaning an interchange in the roles of the electric and magnetic fields. Instead of magnetic charge confined by a magnetic flux tube in a condensate of electrically charged objects (Cooper pairs), the idea is that color electrically charged objects (quarks) are confined by an electric flux tube in a condensate of magnetically charged objects (magnetic monopoles).

The identification of magnetic monopoles in a non-abelian gauge theory requires the selection of an abelian subgroup of the gauge group. In a theory with a Higgs field  $\phi$  in the adjoint representation of the gauge group, such as the Georgi-Glashow model, an expectation



value  $\langle \phi \rangle \neq 0$  breaks the  $SU(N)$  symmetry to an abelian  $U(1)^{N-1}$  subgroup.<sup>‡</sup> By fixing to a unitary gauge, so that  $\langle \phi \rangle$  has some definite direction in the Lie algebra, gauge transformations in the abelian subgroup leave this direction unchanged. Magnetic monopoles can then be identified from the abelian magnetic field associated with the gauge bosons of the abelian subgroup. In QCD there is no Higgs field, but 't Hooft [16] proposed that a composite gluonic operator, transforming like a Higgs field in the adjoint representation of the gauge group, could also serve the purpose of singling out an abelian subgroup.

Numerical studies of the monopole mechanism have gone in two directions. The first, pioneered by the Kanazawa group [17], emphasizes a particular composite operator, or, equivalently, a particular gauge which leaves a remaining  $U(1)^{N-1}$  gauge symmetry. This gauge is the maximal abelian gauge, and on the lattice it is the gauge which makes link variables as diagonal as possible; *e.g.* in  $SU(2)$  gauge theory, the object is to maximize

$$R = \sum_{x,\mu} \text{Tr}[U_\mu(x) \sigma_3 U_\mu^\dagger(x) \sigma_3] \quad (18)$$

where  $\sigma_a$  denote the Pauli matrices. The lattice link variables, which take values in the full  $SU(N)$  group, are then projected to the  $U(1)^{N-1}$  subgroup; this procedure is known as “abelian projection”. One can then identify monopole worldlines in the abelian projected lattice, and check to see if this monopole content gives the correct string tension and other static properties of QCD. The idea is successful in a number of respects, the main difficulty is representation dependence: the force between quarks depends on their  $U(1)^{N-1}$  electric charges, rather than their N-ality [18].

Another approach, that has been developed largely by the Pisa group [19], is to define a monopole creation operator  $\phi^M(x)$  in  $SU(N)$  lattice gauge theory, with the monopoles again defined in some gauge, and check to see that  $\langle \phi^M \rangle \neq 0$ . In an ordinary abelian Higgs theory, a vacuum expectation value  $\langle \phi \rangle \neq 0$  breaks the  $U(1)$  gauge symmetry of the theory, and we have an electric superconductor. The idea is that in a non-abelian gauge theory with no elementary Higgs fields, an expectation value  $\langle \phi^M \rangle \neq 0$  implies the breaking of a dual magnetic symmetry, and confining gauge theories exist in the “dual superconductor” phase.

The idea of dual superconductivity received a great boost from the work of Seiberg and Witten [20], who were able to show analytically that in certain supersymmetric gauge theories, monopole condensation actually does take place. In these theories, unlike QCD, there exists an elementary Higgs field which can be used to single out a unique abelian subgroup; fixing the abelian subgroup by a composite operator is unnecessary. In these particular supersymmetric theories, it is possible to derive explicitly an effective dual abelian Higgs action, at least if the effects of gluons not belonging to the abelian subgroup are ignored. But in the resulting effective theory, as in other “monopole dominance” models [18], the asymptotic string tension between quarks of a given abelian charge depends only on that abelian charge, and not on the quadratic Casimir or the N-ality of the associated  $SU(N)$

<sup>‡</sup> Actually, a local gauge symmetry cannot be spontaneously broken [15], and the distinction between the “broken” or “Higgs” phase, and the unbroken phase, is rather subtle. But the “broken gauge symmetry” terminology is common, even if not strictly correct, and we will continue to use that terminology here.

representation. Related to this fact is a certain multiplicity of Regge trajectories [21], which is also rather unlike QCD.

Monopoles also arise in investigations of objects known as “calorons”, which are instantons at finite temperatures. Instantons are semi-classical solutions of the Euclidean-time gauge field equations, and finite temperature corresponds to a finite, periodic extension in the time direction. Recent studies [22] center on a type of caloron solution with non-trivial holonomy found by Kraan and van Baal [10], and by Lee and Lu [11]. The calorons can be thought of as bound states of monopoles, which tend to move apart from one another as the temperature is lowered [23]. It has been suggested that confinement could be attributed to caloron dynamics. This again raises the question of how a caloron-based confinement mechanism would obtain the correct N-ality dependence of the string tension extracted from Wilson loops (see, however, ref. [24]).

### *3.2. Center Vortices*

We have already mentioned that the asymptotic string tension depends only on the N-ality of the quark charge. This fact is important, and easily understood. First of all, there are an infinite number of  $SU(N)$  representations, but only a finite number of representations of the  $Z_N$  subgroup, so the representations of  $SU(N)$  can be divided into classes, each with the same N-ality. Gluons have N-ality equal to zero. This means that when gluons bind to a color charge in some representation  $r$ , the resulting bound state is in a color representation  $r'$  with the same N-ality as  $r$ . So as a quark in representation  $r$  separates from its antiquark, gluons can be pair-created out of the vacuum, and bind to the quark and antiquark, reducing the color charge of each. However, the color charge can only be reduced to the lowest dimensional representation with the same N-ality as  $r$ . For example, in  $SU(2)$  gauge theory the center group is  $Z_2$ , and representations can be divided into  $j = \text{half-integer}$ , with N-ality one, and  $j = \text{integer}$ , with N-ality zero. As two quarks in the  $j = 3/2$  representation separate, pair-created gluons can bind to the quarks reducing the color charge to  $j = 1/2$ . So the asymptotic string tension of heavy  $j = 3/2$  quarks is the same as that of  $j = 1/2$  quarks, and in fact, by the same argument, the asymptotic string tension of all N-ality=1 quarks are the same. Likewise, two quarks in the adjoint ( $j = 1$ ) representation can bind to gluons, forming two color singlets. The asymptotic string tension of  $j = 1$  quarks is zero, as is the string tension of quarks in any N-ality=0 representation.

It is helpful to think about this N-ality dependence in the context of the Euclidean functional integral over field configurations. In a Monte Carlo simulation, Wilson loops are simply averaged over some finite set of lattice configurations, generated stochastically with the appropriate probability weighting. How do these configurations manage to produce asymptotic string tensions that depend solely on the N-ality of the group representation of the Wilson loop?

The answer to this question comes from an interesting direction. In 1978 't Hooft [25] introduced a loop operator  $B(C)$  in  $SU(N)$  gauge theories, intended to be in some sense “dual”

to the ordinary Wilson loop operator  $W(C)$ , with the rather beautiful commutation property

$$B(C)W(C') = e^{2\pi i/N} W(C')B(C) \quad (19)$$

if curves  $C$  and  $C'$  are topologically linked to one another. 't Hooft argued, just from this commutation relation and the presumed existence of a mass gap, that  $\langle B(C) \rangle$  has a perimeter-law falloff  $\exp(-\mu P(C))$  in the confined phase of a gauge theory, and an area-law falloff in the non-confining Higgs phase. This is precisely opposite to the behavior of Wilson loops in those two phases. It turns out that  $B(C)$  is the creation operator for an object known as a “center vortex”. Roughly speaking, a center vortex is a tube of magnetic flux, in which the exponential of the vortex flux, measured by a Wilson loop  $P \exp[i \oint_{C'} dx^\mu A_\mu]$  running around the vortex, takes values in the  $Z_N$  center of the  $SU(N)$  gauge group. More precisely, creation of a center vortex which is topologically linked to a given Wilson loop changes the Wilson loop by a multiplicative factor equal to a center element. A Wilson loop in some group representation  $r$  is multiplied by a factor  $\exp[2\pi i k/N] \in Z_N$  which depends only on the N-ality  $k$  of the representation  $r$ . This is the crucial property. In order to have a confinement mechanism in which the string tension of a Wilson loop depends only on the N-ality of the loop, it is necessary to have configurations which affect loops of the same N-ality in the same way. Center vortices are the only known field configurations which satisfy this requirement. In D=4 Euclidean dimensions these objects are actually surfaces; they may be thought of as having been swept out by a magnetic vortex loop as it propagates in time. In D=3 dimensions loops can be topologically linked to other loops; in D=4 dimensions loops link to surfaces.

The center vortex confinement mechanism, as elaborated in refs. [25, 26] is quite simple: Center vortices percolate throughout the vacuum, and Wilson loops derive an area law from random fluctuations in the number of vortices piercing the loop. This proposal lay dormant for about fifteen years, but was revived after a series of numerical investigations [27, 28, 29] turned up rather strong evidence in its favor. This evidence is reviewed in detail in ref. [12]. Briefly, the numerical techniques are very close to those employed in abelian projection. Lattice configurations are fixed to a gauge which leaves a residual  $Z_N$  invariance, and  $SU(N)$  group-valued link variables are then projected to the nearest element of the  $Z_N$  subgroup. The excitations of the projected configurations are thin center vortex configurations known as “P-vortices”, and these thin vortex sheets appear to lie within thick vortex surfaces in the unprojected gauge theory. Although gauge fixing is used in the identification, P-vortices correlate with both the gauge-invariant action density and gauge-invariant Wilson loops on the unprojected lattice. P-vortex areas scale with lattice coupling as expected from asymptotic freedom, and by themselves produce an area law falloff for Wilson loops with roughly the right string tension. When vortices are removed from the original lattice, the string tension drops to zero, topological charge vanishes, and chiral symmetry is unbroken.

It is worth expanding a little on vortex removal. Let  $U(C) \in SU(N)$  be a Wilson loop around curve  $C$ , and let  $Z(C) \in Z_N$  be the value of the Wilson loop in the projected configuration. In  $SU(2)$  gauge theory,  $Z(C) = (-1)^{l(C)}$ , where  $l(C)$  is the number of P-vortices linked to the loop. Denote the corresponding Wilson loop in the vortex-removed

configuration by  $U'(C)$ . These expressions have a simple relationship to one another

$$U'(C) = Z(C)U(C) = (-1)^{l(C)}U(C). \quad (20)$$

Numerically, it is found that the projected and unprojected Wilson loop expectation values,  $\langle Z(C) \rangle$  and  $\langle \text{Tr}[U(C)] \rangle$  respectively, both have area-law falloffs with approximately the same string tension, while the string tension of Wilson loops in the vortex-removed configuration  $\langle \text{Tr}[U'(C)] \rangle$  is vanishing. The only way that this can happen, in view of (20), is that the fluctuations in the sign of  $\langle \text{Tr}[U(C)] \rangle$  are correlated to fluctuations in P-vortex linking number. This correlation of the linking number of P-vortices with the sign of gauge-invariant Wilson loops, while certainly not a proof of the vortex confinement mechanism, argues strongly in its favor.

It is also found that the monopole worldlines of the abelian projected lattice lie on P-vortex worldsheets [18]. A center vortex can, in fact, be thought at any given time as a kind of monopole-antimonopole chain, in which the abelian magnetic flux of the monopoles and antimonopoles is collimated along the vortex line. This means that the vortex and abelian monopole pictures are not really antagonistic; the collimation of the monopole flux is exactly what is required for the monopole picture to satisfy the required N-ality dependence for the asymptotic string tension. We also note that Casimir scaling, in the vortex picture, is due to the finite thickness of center vortices [30], and spatial variations of flux within the vortex core [31]. In a gauge theory based on the group  $G(2)$ , which has a trivial center subgroup, the prediction of the vortex theory is that the asymptotic string tension is zero. This agrees with expectations, since in  $G(2)$  gauge theory gluons can combine with quark charges in any representation to form a color singlet.

The main reservations to the vortex picture are numerical: the string tensions in the projected lattices are not quite the same as for the unprojected lattice, and there are concerns regarding the gauge-fixing procedure, which is plagued by Gribov copies [32].

### 3.3. Coulomb Energy and the Gribov Horizon

By definition gauge invariance implies a redundancy in the degrees of freedom of a gauge field. Hamiltonian dynamics requires an elimination of this redundancy via a gauge choice, resulting in a formulation involving the correct number of physical degrees of freedom. In Coulomb gauge, in particular, there is a very suggestive separation between the electric energy due to the longitudinal (*i.e.* Coulombic) and transverse electric fields. Classically, the Coulomb gauge Hamiltonian has the form<sup>S</sup>

$$H = \frac{1}{2} \int d^3x (\vec{E}^{a,tr} \cdot \vec{E}^{a,tr} + \vec{B}^a \cdot \vec{B}^a) + \frac{1}{2} \int d^3x d^3y \rho^a(x) K^{ab}(x,y) \rho^b(y) \quad (21)$$

where  $\rho^a$  is the (matter plus gauge field) color charge density,  $\vec{E}^{a,tr}$  is the transverse color electric field operator, and

$$K^{ab}(x,y) = [M^{-1}(-\nabla^2)M^{-1}]_{x,y}^{ab} \quad (22)$$

<sup>S</sup> Quantum-mechanically there are some operator-ordering modifications, which we will not discuss here.

is the instantaneous Coulomb propagator. The Coulomb interaction energy is given by the non-local term in the Hamiltonian, involving  $\rho K \rho$ . In an abelian theory,  $K(x, y)$  is simply proportional to  $1/|\vec{x} - \vec{y}|$ . In a non-abelian theory,  $K^{ab}(x, y)$  is dependent on the gauge-field through the Faddeev-Popov operator

$$M^{ac} = -\partial_i D_i^{ac}(A) = -\nabla^2 \delta^{ac} - \epsilon^{abc} A_i^b \partial_i. \quad (23)$$

Could it be that the vacuum expectation value of the Coulomb energy, for static sources, leads to an asymptotically linear, rather than  $1/r$ , potential?

Gribov [33] and Zwanziger [34] have argued in favor of this possibility. The argument is roughly as follows: In Coulomb gauge, the integration over gauge fields needs be restricted to  $A$ -fields satisfying  $\nabla \cdot A = 0$ , and for which the Faddeev-Popov operator is positive; *i.e.* for which the eigenvalues of the  $M$  operator are all positive. This positivity condition restricts the gauge fields to a subspace of gauge-field configuration space; the boundary of this region, where the  $M$  operator develops a zero eigenvalue, is known as the “Gribov horizon”. Since the Coulomb propagator  $K^{ab}(x, y)$  depends on the inverse of  $M$  this operator becomes very large in the neighborhood of the Gribov horizon. Now, since the dimension of configuration space is very large, it is reasonable that the bulk of configurations are located close to the horizon (just as the volume measure  $r^{d-1} dr$  of a ball in  $d$ -dimensions is sharply peaked near the radius of the ball). Since it is the inverse of the  $M$  operator which appears in the Coulomb energy, it is possible that the near-zero eigenvalues of this operator will enhance the magnitude of the energy at large quark separations, possibly resulting in a confining potential at large distances. Moreover, it is possible to prove the following inequality [35]: If  $V(R)$  is the static quark potential (*i.e.* the minimal energy of a physical state containing two static quark-antiquark sources in the fundamental representation), and  $V_{coul}(R)$  is the Coulomb potential obtained from the vacuum expectation value  $\langle \rho K \rho \rangle$ , then

$$V(R) \leq V_{coul}(R). \quad (24)$$

This means that “Coulomb confinement” is a necessary (but not sufficient) condition for confinement. The interesting question is whether  $V_{coul}(R)$  is linear and, if linear, whether the corresponding string tension  $\sigma_{coul}$  equals the string tension of the static quark potential.

These questions have been investigated numerically, via lattice Monte Carlo in Coulomb gauge. The answer is that  $V_{coul}(R)$  does indeed rise linearly [36, 37]. Moreover, the infrared divergent Coulombic energy of an isolated charge comes about by precisely the mechanism suggested by Gribov and Zwanziger: a large density of eigenvalues of the Faddeev-Popov operator in the neighborhood of the zero eigenvalue [38]. When center vortices are removed from lattice configurations, the Coulombic energy is non-confining, and the Faddeev-Popov eigenvalue distribution resembles that of the abelian theory. Together with the fact that center vortices are field configurations lying on the Gribov horizon, this suggests a close relationship between these two confinement scenarios.

On the other hand, it also turns out that the Coulomb string tension is about three times larger than the string tension of the static quark potential, so the behavior of  $K(x, y)$  cannot be the whole story behind confinement. There is, in fact, an even more basic objection: Any theory of confinement based on one-gluon (or one quasi-particle) exchange will have

difficulties in explaining why the color electric field is collimated into a flux tube. In general, one-particle exchange models of the confining force give rise to long-range dipole fields. If a proton, say, were held together by one-gluon exchange forces, it is hard to see why there could not be long-range color van-der-Waals forces among distant protons, contrary to observation.

One interesting approach, followed in ref. [39], is to try to construct physical states in Coulomb gauge, whose energy is lower than that of a quark-antiquark pair plus their Coulomb field, by adding constituent gluons. Possibly this could also help with the problem of the long-range dipole field. We might imagine that as a quark and antiquark separate, they pull out between them a “chain” of constituent gluons, with each gluon in the chain bound to its nearest neighbors by Coulombic forces. This is the “gluon chain model” [40], and it provides a picture of the QCD flux tube as a kind of discretized string. Whether this gluon-chain picture will eventually emerge from investigations in Coulomb gauge or, alternatively, from a recent worldsheet formulation of gauge theory quantized in light-cone gauge [41], remains to be seen.

Before leaving this topic, we might ask: Given that instantaneous one-gluon exchange results in a linear attractive potential between a quark and an antiquark, what would be situation for two quarks? Would we end up with a finite energy color non-singlet state and a linear repulsive potential? That would, of course, be a real disaster for this approach. In fact, in calculating Coulomb interaction energies of composite states one has to carefully take into account the cancellation of divergences which are encountered in both the quark self-energy and one-gluon exchange terms. It turns out that for color singlets, these divergences precisely cancel, leaving a finite attractive potential, while in non-singlets the divergent self-energies are not cancelled, and the energy of the non-singlet state is infinite. This cancellation is discussed in ref. [42], and demonstrated, in the context of a Bethe-Salpeter approach, in ref. [43] and references therein.

### 3.4. Functional Approaches

Functional approaches employed to the infrared behaviour of QCD are Schwinger-Dyson Equations (SDEs) and Renormalization Group Equations<sup>||</sup>, for a recent review see [45]. In the Landau gauge the analytical treatment of these equations in the far infrared have provided a number of exact inequalities for the infrared exponents of gluon and ghost one-particle irreducible (1PI) Green functions. Gauge fixing is hereby performed by the standard Faddeev-Popov method supplemented by auxiliary conditions such that the generating functional consists of an integral over gauge field configurations that are contained in the first Gribov region. The employed method has been justified using ghost-free stochastic quantisation [46]. The resulting SDEs for 1PI-Green functions have been solved analytically in the infrared to all orders in a skeleton expansion (*i.e.* a loop expansion using full propagators and vertices) [47]. It turns out that these Green’s functions are infrared singular in case *all* external momenta go to zero. A remarkable property of the infrared solution is the fact that it is generated by

<sup>||</sup> A combination of both methods has recently allowed to *uniquely* determine the infrared behavior of all Green functions of Landau gauge Yang-Mills theory [44].

exactly those parts of the SDEs that involve ghost loops. In other words: the Faddeev-Popov determinant dominates the infrared behaviour of non-Abelian Yang-Mills theories. Thus an infrared asymptotic theory can be obtained by ‘quenching’ the Yang-Mills action, *i.e.* setting  $\exp(-S_{YM}) = 1$  in the generating functional [46]. The solution of this asymptotic theory is given by power laws.

The basic examples for power law solutions are the ghost and gluon propagators

$$D^G(p^2) = -\frac{G(p^2)}{p^2}, \quad D_{\mu\nu}(p^2) = \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}. \quad (25)$$

The corresponding power laws in the infrared are

$$G(p^2) \sim (p^2)^{-\kappa}, \quad Z(p^2) \sim (p^2)^{2\kappa}. \quad (26)$$

Since  $\kappa$  is positive [48] one obtains an infrared enhanced ghost and an infrared suppressed gluon propagator. In Landau gauge an explicit value for  $\kappa$  can be derived from the observation that the dressed ghost-gluon vertex becomes (almost) bare in the infrared, one then obtains  $\kappa = (93 - \sqrt{1201})/98 \approx 0.595$  [49, 50].<sup>Φ</sup>

Let us shortly digress here and mention in which sense this leads to the so-called “kinematic confinement” of transverse gluons.<sup>†</sup> First we note that covariant quantum theories of gauge fields require indefinite metric spaces. Abandoning the positivity of the representation space already implies to give up one of the axioms of standard quantum field theory. Maintaining the much stronger principle of locality gluon confinement then naturally relates to the violation of positivity in the gauge field sector, see *e.g.* ref. [55]. Similar to QED, where the Gupta-Bleuler prescription [56] is to enforce the Lorentz condition on physical states, a semi-definite physical subspace can be defined as the kernel of an operator. The physical states then correspond to (equivalence classes of) states in this subspace. Covariance implies, besides transverse photons, the existence of longitudinal and timelike (“scalar”) photons in QED. The latter two form metric partners in the indefinite space: They cancel against each other in every  $S$ -matrix element and therefore do not contribute to observables.

In QCD cancelations of unphysical degrees of freedom in the  $S$ -matrix also occur but are more complicated due to the self-interaction of the gluons. A consistent quantum formulation in a functional integral approach leads to the introduction of ghost fields [57]. The proof of the cancelation of longitudinal and timelike gluons in every  $S$ -matrix element to all orders in perturbation theory has been possible by employing the BRST symmetry [58] of the covariantly gauge fixed theory. At this point one has achieved a consistent quantization.

Based on the BRST formalism and implications of the Gribov horizon [59] positivity violation of the propagator of transverse gluons has been a long-standing conjecture for which there is now compelling evidence, see *e.g.* ref. [60] and references therein. The basic features underlying these gluon properties, namely the infrared suppression of correlations

<sup>Φ</sup> Dynamical quarks do not change the infrared behavior of the gluon and ghost propagators [51].

<sup>†</sup> The mechanism becomes most transparent in a covariant formulation which includes the choice of a covariant gauge, of course. However, the arguments for the positivity violation in the propagator of transverse gluons are analogously applicable in the Coulomb gauge, and numerical evidence for it is equally firm as in the Landau gauge case, see *e.g.* refs. [34, 52, 53, 54, 37].

of transverse gluons and the infrared enhancement of ghost correlations, has been verified in quite a number of lattice Monte-Carlo calculations and different functional approaches. One then concludes that transverse gluons possess metric partners, they form a so-called BRST quartet together with gluon-ghost, gluon-antighost and gluon-ghost-antighost states. Gluon confinement then occurs as necessarily complete cancelation between amplitudes (Feynman diagrams) containing these states as asymptotic states (as external lines). This is in line with the naïve interpretation of a gluon propagator which vanishes in the infrared: A zero in the propagator at  $p^2 = 0$  implies that there is no propagation of gluons at long distances.<sup>‡</sup>

An additional important consequence of this infrared solution for gluons and ghosts is the qualitative universality of the running coupling in the infrared. Renormalisation group invariant couplings can be defined from either of the primitively divergent vertices of Yang-Mills-theory, *i.e.* from the ghost-gluon vertex, the three-gluon vertex, or the four-gluon vertex. All three couplings approach a fixed point in the infrared. However, the explicit value of the fixed point may be different for each coupling. For a bare ghost-gluon vertex one obtains  $\alpha^{gh-gl}(0) \approx 8.92/N_c$  [49, 62]; the other couplings have not been determined yet.<sup>S</sup> This behavior sheds light on the existence of the power law solutions: pure Yang-Mills theory becomes approximately conformal in the far infrared.

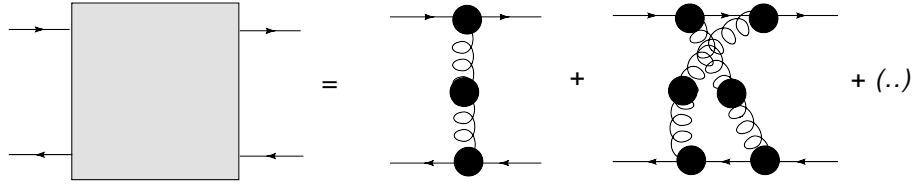
As explained in the introduction the static quark potential is a property of would-be infinitely heavy quarks. To extend the infrared analysis to full QCD [63] one concentrates first on the quark sector of quenched QCD and chooses the masses of the valence quarks to be large, *i.e.*  $m > \Lambda_{\text{QCD}}$ . The remaining scales below  $\Lambda_{\text{QCD}}$  are those of the external momenta of the Green functions. Without loss of generality these can be chosen to be equal, since infrared singularities in the corresponding loop integrals appear only when all external scales go to zero [47]. One can then employ SDEs to determine the selfconsistent solutions in terms of powers of the small external momentum scale  $p^2 \ll \Lambda_{\text{QCD}}$ . The SDEs which have to be considered in addition to the SDEs of Yang-Mills theory are the one for the quark propagator and the quark-gluon vertex. The dressed quark-gluon vertex  $\Gamma_\mu$  consists in general of twelve Dirac tensor structures. Some of these tensor structures would have to vanish if chiral symmetry would not be broken (either explicitly or dynamically). Especially those Dirac-scalar structures are, in the chiral limit, generated non-perturbatively together with the dynamical quark mass function in a self-consistent fashion: Dynamical chiral symmetry breaking reflects itself thus not only in the propagator but also in a three-point function.

An infrared analysis of the full set of DSEs reveals that a solution exists such that vector and *scalar* components of the quark-gluon vertex are infrared divergent with an exponent related to  $\kappa$  [63]. A numerical solution of truncated set of SDEs confirms this infrared behavior. Similar to the Yang-Mills sector it is the diagram containing the ghost loop that dominates. Thus all effects from the Yang-Mills sector are generated by the infrared asymptotic theory described above. More importantly, in the quark sector the driving pieces of this solution is the scalar Dirac amplitude of the quark-gluon vertex and the scalar part of

<sup>‡</sup> For chromomagnetic gluons this picture persists in the high-temperature phase of QCD [61].

<sup>S</sup> For all these couplings the infrared fixed point behaves like  $1/N_c$  thus obeying the correct large- $N_c$  behavior for all values of  $N_c$ . A detailed investigation of the large- $N_c$  limit of this approach is, however, still lacking.





**Figure 3.** The four-quark 1PI Green's function and the first terms of its skeleton expansion, adapted from ref. [47].

the quark propagator. Both pieces are only present when chiral symmetry is broken.

The static quark potential is obtained from the four-quark 1PI Green's function  $H(p)$ , which is given in Fig. 3 together with its skeleton expansion. From the infrared analysis one infers that  $H(p) \sim (p^2)^{-2}$  in the infrared. From the usual relation

$$V(\mathbf{r}) = \int \frac{d^3 p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{p}\mathbf{r}} \sim |\mathbf{r}| \quad (27)$$

between the static four-quark function  $H(p^0 = 0, \mathbf{p})$  and the quark potential  $V(\mathbf{r})$  one therefore obtains a linear rising potential. Correspondingly, the running coupling from the quark-gluon vertex turns out to be proportional to  $1/p^2$  in the infrared, *i.e.* contrary to the couplings from the Yang-Mills vertices this coupling is singular in the infrared.

Already the first term in the skeleton expansion, *i.e.* the effective, nonperturbative one-gluon exchange displayed in Fig. 3, generates this result. Since the following terms in the expansion are equally enhanced in the infrared, the string tension will be built up by summing over an infinite number of diagrams. The latter property is bad news for the usefulness of the approach but it had to be expected in the first place. Since already an effective, nonperturbative one-gluon exchange generates the confining potential one is again, as in the previous subsection, confronted with the problem of unwanted van-der-Waals forces. The suppressed gluon propagator looks at first sight helpful because it implies that there are no long-range correlations between the gluons, *i.e.* the gauge fields, and thus for chromoelectric and chromomagnetic fields at large distances. However, the problem of avoiding long-range multipole fields has only been shifted from the two-point correlation to a specific three-point function, namely the quark-gluon vertex. In addition, as very likely every picture based on a finite number of quasi-particles has to fail in explaining the Lüscher term (17) one can already conclude that the series in Fig. 3 needs to be an infinite one if the picture were able to describe quark confinement correctly.

Last but not least, N-ality can occur in such pictures only if cancelations as *e.g.* between gluons and adjoint quarks will take place. Casimir scaling, on the other hand, requires that at intermediate distances these cancelations are still absent or incomplete. Explaining these features of confinement is still a completely unsolved challenge within functional approaches.

### 3.5. AdS/CFT correspondence

There is compelling evidence that a type-IIB closed superstring theory in ten dimensions is dual to an  $\mathcal{N} = 4$  super-Yang-Mills (sYM) theory. The space-time in the superstring theory is

such that five dimensions form a sphere, and the other five dimensions a non-compact anti-de Sitter space, briefly denoted by  $\text{AdS}_5$ . Hereby the sphere has a (positive) radius  $R$  and  $\text{AdS}_5$  a negative curvature of the same scale

$$ds^2 = R^2 d\Omega_5^2 + \frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu. \quad (28)$$

The  $\mathcal{N} = 4$  sYM theory has as much supersymmetry a gauge theory can have: It is a conformal field theory (CFT). Mathematically, the duality is built on the fact that the isometry group of  $\text{AdS}_5$  is isomorphic to the conformal group of four-dimensional Minkowski space,  $\text{SO}(4,2)$ .<sup>||</sup> Employing a suitable background metric the gauge coupling is inversely proportional to the string tension:

$$g^2 N_c = R^4 / \alpha'^2. \quad (29)$$

This implies that the strong coupling gauge theory is dual to weak coupling string theory, and thus, as the low-energy limit of superstring theory is supergravity, to weak coupling gravity.

Real-world QCD is not supersymmetric. Therefore one needs to break supersymmetry and therefore also conformal invariance. The corresponding models typically modify the metric in the infrared by introducing cutoffs, or equivalently black hole type backgrounds. The corresponding minimal distance is then identified with the inverse of the QCD scale:<sup>Φ</sup>

$$r_{\min} = 1/\Lambda_{\text{QCD}}. \quad (30)$$

In those black hole metrics the minimal surface spanning a Wilson loop of increasing size eventually has to approach  $r = r_{\min}$ . Beyond this point no red shift factor contributes to the area of the surface, it grows proportional to  $R^2 r_{\min}^2$  providing a non-vanishing string tension

$$\sigma = 1/2\pi\alpha'_{\text{QCD}} = R^2 r_{\min}^2 / 2\pi\alpha' = \sqrt{2g^2 N_c} / 2\pi\Lambda_{\text{QCD}}^2. \quad (31)$$

Another approach is to introduce so-called “fractional D-branes” to break the supersymmetry and conformal invariance, *c.f.* ref. [64].

Since the first considerations of Wilson loops within AdS/CFT correspondence, see *e.g.* ref. [65], a large number of papers appeared on the subject, and a summary of these developments is far beyond the scope of the present article. We nevertheless want to note that the N-ality condition has recently been shown to be fulfilled in this approach [66].

Phenomenological tests of AdS/CFT correspondence are abundant, it has especially been successful in reproducing general properties of scattering processes of QCD bound states. Hereby confinement can be simulated by cutting off the extension of hadron wave function into the “fifth” dimension [67]. The interested reader can obtain a first impression from refs. [68], the references therein provide a reasonable guide for further reading.

<sup>||</sup> It is an irony of this field that this first example of AdS/CFT duality does not confine because  $\mathcal{N} = 4$  sYM theory is exactly conformal. When a large Wilson loop is introduced on the boundary of  $\text{AdS}_5$  the red shift factor  $r^2/R^2$  allows the minimal surface spanning the loop to stay finite implying a perimeter instead of an area law for the sYM Wilson loop.

<sup>Φ</sup> In most corresponding calculations the strong-coupling limit of the smallest glueball mass is used to set the scale.

It is plainly obvious but it should nevertheless be emphasized here: The AdS/CFT correspondence provides no explanation for confinement. It is a calculational tool relating low-energy, non-perturbative QCD to weak-coupling gravity where the background has been *chosen* such to provide confinement in QCD. However, as some of the related problems can be treated much easier in the gravity language the approach has and likely will furthermore provide insights into the special properties of possible confinement scenarios in QCD.

#### **4. Conclusions**

It is odd to have a complete theory of one of the four well-established forces of nature – the strong nuclear force – and still not have general agreement, after more than thirty years of effort, on how that force really works at long distances. Nevertheless, there *has* been appreciable progress in this subject, much of it aided by computer simulations. First of all there is a better appreciation of the general features of the confining force, e.g. the color group representation dependencies (N-ality, Casimir scaling) of the confining potential, and the existence and string-like properties of the color electric flux tube, which constrain possible explanations of confinement. Secondly, there exist a reasonable set of suggestions about the origin of confinement, some dating back to the late 1970's and some much more recent, which have, over the last decade, received substantial support from lattice Monte Carlo simulations. It has turned out that a number of these suggestions are related in interesting ways: monopole worldlines, essential to dual-superconductor scenarios, are found to lie on center vortex worldsheets, and center vortex worldsheets appear to be crucial in some ways to the confinement scenario in Coulomb gauge. Both Coulomb and Landau gauge investigations emphasize the importance of the Faddeev-Popov operator, and the infrared properties of the ghost propagator.

There are other proposals for the confinement mechanism which we have not included here. It is impossible to provide an exhaustive discussion in a short article, so we have concentrated on those proposals which, in our judgement, seem best supported by existing numerical studies or other arguments. But it is certainly not excluded that progress may come from some quite different direction.

The confinement problem, in our view, is one of the truly fundamental problems in physics. Quark confinement is the essential link between the microscopic quark-gluon degrees of freedom of QCD, and the actual strong-interaction spectrum of color-neutral mesons, baryons, and nuclei. Until this phenomenon is well understood, something essential is still lacking in our grasp of the foundations of nuclear physics, and the deeper mechanisms of non-abelian gauge theory. Although the confinement problem is hard, the solution is important, and well worth pursuing.

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